Proceedings of
the ASME 2010 Summer Bioengineering Conference
August 30-September 2, 2010, Naples, Florida, USA

A FIRST OPTIMAL CONTROL SOLUTION FOR A COMPLEX, NONLINEAR, TENDON DRIVEN NEUROMUSCULAR FINGER MODEL

Evangelos A. Theodorou*
Department of Computer Science
University of Southern California
Los Angeles, California 90089
Email: etodorov@gmail.com

Emo Todorov
Department of Computer Science
and Engineering
University Washington
Seattle,Washington
Email: todorov@cs.washington.edu

Francisco J. Valero-Cuevas
Department of Biomedical Engineering
University of Southern California
Los Angeles, California, 90089
Email: valero@usc.edu

ABSTRACT

In this work we present the first constrained stochastic optimal feedback controller applied to a fully nonlinear, tendon driven index finger model. Our model also takes into account an extensor mechanism, and muscle force-length and force-velocity properties. We show this feedback controller is robust to noise and perturbations to the dynamics, while successfully handling the nonlinearities and high dimensionality of the system. By extending prior methods, we are able to approximate physiological realism by ensuring positivity of neural commands and tendon tensions at all times.

INTRODUCTION

Stochastic optimal control theory is considered as a valuable tool to understand neuromuscular behavior [1–3]. In contrast, other approaches to neuromuscular control require target time histories of limb kinematics, kinetics and/or muscle activity. See [4] for a review. The application of the dominant optimal control methods such *Linear Quadratic Regulator* (LQR) and *Linear Quadratic Gaussian* (LQG) control, however, has been limited to cases of linear dynamical systems. This cannot capture the nonlinear behavior of muscles and multi-body limbs. In [5], an *Iterative Linear Quadratic Regulator* (iLQG) was introduced to allow the optimal control of nonlinear neuromuscular models. Until now, the iLQG formulation has not been scaled up or applied to high-dimensional neuromuscular plants. Here we extend the iLQG formulation to be applicable in these conditions and thus can, for the first time, use the optimal control framework to predict biologically plausible tendon tensions for a nonlinear neuromuscular finger model.

METHODS

1 Muscle Model

The rigid-body triple pendulum finger model with slightly viscous joints is actuated by Hill-type muscle models. Joint torques are generated by the seven muscles of the index finger (FDP, FDS, EI, EC, LUM, DI, PI), whose tendons act via a simulated extensor mechanism [6]. The resulting torques are formulated as $\tau = \mathbf{M}(\theta) \cdot \mathbf{T}\left(\alpha, L(\theta), V\left(\theta, \dot{\theta}\right)\right)$ where $\mathbf{M}(\theta)$ is the moment arm matrix. The tension $\mathbf{T}\left(\alpha, L(\theta), V\left(\theta, \dot{\theta}\right)\right)$ depends on the activation of the muscles but also varies as a function of muscle velocity $V = V(\theta, \dot{\theta})$ and muscle length $L = L(\theta)$. Therefore the tension is mathematically formulated as $\mathbf{T}\left(\alpha, L(\theta), V\left(\theta, \dot{\theta}\right)\right) = F_L(L(\theta)) \cdot F_V\left(V\left(\theta, \dot{\theta}\right)\right) \cdot \alpha + F_P(L(\theta))$ where the terms $F_L(L(\theta))$, $F_V\left(V\left(\theta, \dot{\theta}\right)\right)$ and $F_P(L(\theta))$ are force functions that describe the force - length and force-velocity properties of muscles [7].

2 Constrained Iterative Stochastic Optimal Control

We consider the class of stochastic optimal control problem with state and control constraints. Such optimal control problems are formulated as

$$v^{\pi}(\mathbf{x}, t) = \min E\left(h(\mathbf{x}(T)) + \int_{t_0}^{T} \ell\left(\tau, \mathbf{x}(\tau), \pi(\tau, \mathbf{x}(\tau))\right) d\tau\right)$$

*Address all correspondence to this author.

subject to $\mathbf{dx} = f(\mathbf{x},\mathbf{u})dt + F(\mathbf{x},\mathbf{u})\mathbf{d}\omega$ and constrains $\phi(\mathbf{x},\mathbf{u}) < 0$, $\psi(\mathbf{x},\mathbf{u}) = 0$. The variable $\mathbf{x} \in \Re^{n\times 1}$ is the state, $\mathbf{u} \in \Re^{m\times 1}$ is the control and $\omega \in \Re^{p\times 1}$ is Brownian noise with variance $\sigma^2 I_{p\times p}$. The stochastic differential equation above corresponds to a rather general class of dynamical systems which are found in robotics and biomechanics. The term $h(\mathbf{x}(T))$ is the terminal cost in the cost function while the $\ell(\tau,\mathbf{x}(\tau),\pi(\tau,\mathbf{x}(\tau)))$ is the instantaneous cost rate which is a function of the state $\mathbf{x}$ and control policy $\pi(\tau,\mathbf{x}(\tau))$. The quantity $v^{\pi}(\mathbf{x},t)$ is the cost to go. In the proposed algorithm, the dynamics and the state and control constrains are linearly approximated while the cost is expanded up to the quadratic term. More precisely the deterministic dynamics are first discretized and thus we have $\bar{\mathbf{x}}_{t_{k+1}} = \bar{\mathbf{x}}_{t_k} + \Delta t f(\bar{\mathbf{x}}_{t_k},\bar{\mathbf{u}}_{t_k})$. The resulting discrete time dynamics are linearized around $\bar{\mathbf{x}}_{t_k}$ according to $\delta\mathbf{x}_{t_{k+1}} = A_k\mathbf{x}_{t_k} + B_k\delta\mathbf{u}_{t_k} + \Gamma_k\left(\delta\mathbf{u}_{t_k}\right)\xi_{t_k}$ with $A_k = I + dt\partial\mathbf{f}/\partial\mathbf{x}$ and $B_k = dt\partial\mathbf{f}/\partial\mathbf{u}$ the state and control transition matrices and Γ_k the noise transition matrix that is control depended. The quadratic approximation of the cost function is given as $Cost_k = q_k + \delta\mathbf{x}_{t_k}^T\mathbf{q} + \frac{1}{2}\delta\mathbf{x}_{t_k}^T Q_k\mathbf{x}_{t_k} + \delta\mathbf{u}_{t_k}^T\mathbf{r} + \frac{1}{2}\delta\mathbf{u}_{t_k}^T R_k\mathbf{u}_{t_k} + \delta\mathbf{x}_{t_k}^T P_k\mathbf{u}_{t_k}$. Since the cost to go is quadratically approximated we will have $v_k(\delta\mathbf{x}) = s_k + \mathbf{s}_{k+1}^T\delta\mathbf{x} + \delta\mathbf{x}^T S_{k+1}\delta\mathbf{x}$ where the terms $s_k, \mathbf{s}_{k+1}$ and S_{k+1} are backward propagated from the terminal state. Every iteration i of the iterative optimal control algorithm consists of:

1. A backward pass of the cost to go function $(S_k,\mathbf{s}_k,s_k)$
2. Calculation of the control variations $\delta\mathbf{u}_{t,\ldots,T}^{(i)}$ and update of the controls $\mathbf{u}_{t,\ldots,T}^{(i)} = \delta\mathbf{u}_{t,\ldots,T}^{(i)} + \mathbf{u}_{t,\ldots,T}^{(i-1)}$.
3. A forward pass of the dynamics $\bar{\mathbf{x}}_{t_{k+1}} = \bar{\mathbf{x}}_{t_k} + \Delta t f(\bar{\mathbf{x}}_{t_k},\bar{\mathbf{u}}_{t_k}^i)$ to get the new state trajectory $\bar{\mathbf{x}}_{t,\ldots,T}^i$.
4. Calculation of the approximations of dynamics and cost around the state and control trajectories $(\bar{\mathbf{x}}_{t,\ldots,T}^i, \mathbf{u}_{t,\ldots,T}^{(i)})$.

We extended the above formulation by the addition of Lagrange multipliers that enforce active constraints on the states and controls. Minimizing this new adjoint cost function finds the optimal state and non-negative control trajectories. In these example,we only constrain the controls to no negative. The task for the finger is to move downwards to tap a specific point on a surface by arriving with non-zero velocity.

RESULTS AND CONCLUSIONS

The iLQG algorithm with constraints on the controls is able to predict optimal tendon tension time histories on its own, and these time histories are both physiologically realistic and reflect the time varying interactions needed to orchestrate the coordinated finger flexion movement, see figure. We note the importance of the coordinated action among the intrinsic (LUM, DIP and PI), extensor (EIP and EDC) and flexor superficialis (FDS) muscles. The absence of a decelerating burst in the extensors and the relative silence of the flexor digitorum profundus (FDP) is sensitive to moment arm parameters. Future work will establish whether EMG signals measured in human subjects [8] are

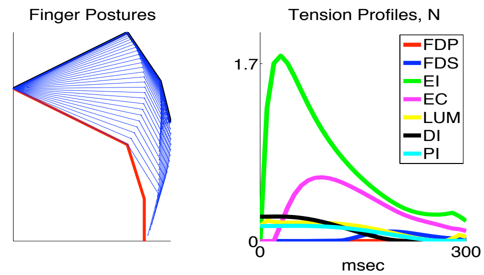

Figure 1. Sequence of postures, final posture in red. Right panel shows tendon tensions. Note all are positive.

compatible with this optimal control policy. Our advances enable the testing of these hypotheses in future work because it demonstrates a first example of the scalability and applicability of the iLQG framework to complex nonlinear neuromuscular systems.

ACKNOWLEDGEMENTS

This material is based upon work supported by NSF Grants EFRI-COPN 0836042, BES-0237258, and NIH Grants AR050520 and AR052345 to FVC.

REFERENCES

[1] He, J., Levine, W. S., and Loeb, G., 1991. "Feedback gains for correcting small perturbations to standing posture". IEEE Transactions on Automatic Control, **36** (3) , pp. 322–332.

[2] Harris, C. M., and Wolpert, D. M., 1998. "Signal-dependent noise determines motor planning". *Nature,* **394** , pp. 780–784.

[3] E.Todorov, 2005. "Stochastic optimal control and estimation methods adapted to the noise characteristics of the sensorimotor system". Neural Computation, **17** (5) , pp. 1084–1108.

[4] Valero-Cuevas, F. J., Hoffmann, H., Kurse, M. U., Kutch, J. J., and Theodorou, E. A., 2009. "Computational Models for Neuromuscular Function". IEEE Reviews in Biomedical Engineering (2) October , pp. 110–135.

[5] Todorov, E., and Li, W., 2005. "A generalized iterative LQG method for locally-optimal feedback control of constrained nonlinear stochastic systems". In Proceedings of the American Control Conference.

[6] Brook, N., J, M., M, S., and J, D., 1995. "A biomechanical model of the index finger dynamics". *Medical Engineering Physics.,* **17** , pp. 54–63.

[7] Brown, I., and Loeb, G., 2000. "Measured and modeled properties of the mammalian skeletal muscle IV. Dynamics of activation and deactivation ". *Journal of Muscle Research and Cell Mobility,* **21** , pp. 33 – 47.

[8] Venkadesan, M., and Valero-Cuevas, F., 2008. " Neural control of motion- to force transitions with the fingertip". The journal of Neuroscience, **28** (6) , pp. 1366–1373.