A FIRST OPTIMAL CONTROL SOLUTION FOR A COMPLEX, NONLINEAR, TENDON
DRIVEN NEUROMUSCULAR FINGER MODEL

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ABSTRACT
In this work we present the first constrained stochastic optim-
tal feedback controller applied to a fully nonlinear, tendon
driven index finger model. Our model also takes into account an
extensor mechanism, and muscle force-length and force-velocity
properties. We show this feedback controller is robust to noise
and perturbations to the dynamics, while successfully handling
the nonlinearities and high dimensionality of the system. By ex-
tending prior methods, we are able to approximate physiological
realism by ensuring positivity of neural commands and tendon
tensions at all times.

INTRODUCTION
Stochastic optimal control theory is considered as a valu-
able tool to understand neuromuscular behavior [1–3]. In con-
trast, other approaches to neuromuscular control require target
time histories of limb kinematics, kinetics and/or muscle activ-
ity. See [4] for a review. The application of the dominant optimal
control methods such Linear Quadratic Regulator (LQR) and
Linear Quadratic Gaussian (LQG) control, however, has been
limited to cases of linear dynamical systems. This cannot capture
the nonlinear behavior of muscles and multi-body limbs. In [5],
an Iterative Linear Quadratic Regulator (iLQG) was introduced
to allow the optimal control of nonlinear neuromuscular models.
Until now, the iLQG formulation has not been scaled up or ap-
plied to high-dimensional neuromuscular plants. Here we extend
the iLQG formulation to be applicable in these conditions and
thus can, for the first time, use the optimal control framework
to predict biologically plausible tendon tensions for a nonlinear
neuromuscular finger model.

METHODS
1 Muscle Model
The rigid-body triple pendulum finger model with slightly
viscous joints is actuated by Hill-type muscle models. Joint
torques are generated by the seven muscles of the index fin-
ger (FDP, FDS, EI, EC, LUM, DI, PI), whose tendons act via a
simulated extensor mechanism [6]. The resulting torques are for-
mulated as
\[ \tau = M(\theta) \cdot T (\alpha, L(\theta), V(\theta, \dot{\theta})) \]
where \( M(\theta) \) is the moment arm matrix. The tension \( T (\alpha, L(\theta), V(\theta, \dot{\theta})) \) depends on the activation of the muscles but also varies as a func-
tion of muscle velocity \( V = V(\theta, \dot{\theta}) \) and muscle length \( L = L(\theta) \). Therefore the tension is mathematically formulated as
\[ T (\alpha, L(\theta), V(\theta, \dot{\theta})) = F_L(L(\theta)) \cdot F_V(V(\theta, \dot{\theta})) \cdot \alpha + F_P(L(\theta)) \]
where the terms \( F_L(L(\theta)) \), \( F_V(V(\theta, \dot{\theta})) \) and \( F_P(L(\theta)) \) are force functions that describe the force-length and force-velocity prop-
erties of muscles [7].

2 Constrained Iterative Stochastic Optimal Control
We consider the class of stochastic optimal control problem
with state and control constraints. Such optimal control problems
are formulated as
\[ v^* (x, t) = \min E \left( h(x(T)) + \int_{t_0}^T \ell (\tau, x(\tau), \pi(\tau, x(\tau))) d\tau \right) \]
subject to $\mathbf{d}x = f(\mathbf{x}, \mathbf{u}) dt + F(\mathbf{x}, \mathbf{u}) \mathbf{d}o$ and constraints $\phi(\mathbf{x}, \mathbf{u}) < 0$, $\psi(\mathbf{x}, \mathbf{u}) = 0$. The variable $\mathbf{x} \in \mathbb{R}^{n \times 1}$ is the state, $\mathbf{u} \in \mathbb{R}^{m \times 1}$ is the control and $\omega \in \mathbb{R}^{p \times 1}$ is Brownian noise with variance $\sigma^2 I_{p \times p}$.

The stochastic differential equation above corresponds to a rather general class of dynamical systems which are found in robotics and biomechanics. The term $h(\mathbf{x}(T))$ is the terminal cost in the cost function while the $\ell(\mathbf{u}(\tau), \mathbf{w}(\tau, \mathbf{x}(\tau)))$ is the instantaneous cost rate which is a function of the state $\mathbf{x}$ and control policy $\pi(\mathbf{x}(\tau))$. The quantity $\mathbf{v}(\mathbf{x}, t)$ is the cost to go. In the proposed algorithm, the dynamics and the state and control constrains are linearly approximated while the cost is expanded up to the quadratic term. More precisely the deterministic dynamics are first discretized and thus we have $\tilde{\mathbf{x}}_{k+1} = \tilde{\mathbf{x}}_k + \Delta t f(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k)$.

The resulting discrete time dynamics are linearized around $\tilde{\mathbf{x}}_k$ according to $\hat{\mathbf{x}}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{G}_k (\mathbf{u}_k) \xi_k$ with $A_k = I + d t \mathbf{f}/\partial \mathbf{x}$ and $B_k = d t \mathbf{f}/\partial \mathbf{u}$ the state and control transition matrices and $\mathbf{G}_k$ the noise transition matrix that is control depended.

The quadratic approximation of the cost function is given as $\text{Cost}_k = q_k + \Delta x^T Q_c \mathbf{x}_k + \frac{1}{2} \Delta u^T R_u \mathbf{u}_k + \Delta x^T R \mathbf{x}_k$.

Since the cost to go is quadratically approximated we will have $v_k(\Delta \mathbf{x}) = s_k + s_k^T \Delta \mathbf{x} + \Delta \mathbf{x}^T S_k \Delta \mathbf{x}$ where the terms $s_k, s_{k+1}$ and $S_k$ are backward propagated from the terminal state. Every iteration $i$ of the iterative optimal control algorithm consists of:

1. A backward pass of the cost to go function $(s_k, s_{k+1}, s_{k+2}, ...)$
2. Calculation of the control variations $\delta \mathbf{u}_i$ and update of the controls $\mathbf{u}_i = \delta \mathbf{u}_i + \mathbf{u}_{i-1}$.
3. A forward pass of the dynamics $\tilde{\mathbf{x}}_{k+1} = \tilde{\mathbf{x}}_k + \Delta t f(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k)$ to get the new state trajectory $\tilde{\mathbf{x}}_i$.
4. Calculation of the approximations of dynamics and cost around the state and control trajectories $(\tilde{\mathbf{x}}_i, \mathbf{u}_i)$.

We extended the above formulation by the addition of Lagrange multipliers that enforce active constraints on the states and controls. Minimizing this adjoint cost function finds the optimal state and non-negative control trajectories. In these examples, we only constrain the controls to no negative. The task for the finger is to move downwards to tap a specific point on a surface by arriving with non-zero velocity.

RESULTS AND CONCLUSIONS

The iLQG algorithm with constraints on the controls is able to predict optimal tendon tension time histories on its own, and these time histories are both physiologically realistic and reflect the time varying interactions needed to orchestrate the coordinated finger flexion movement, see figure. We note the importance of the coordinated action among the intrinsic (LUM, DIP and PI), extensor (EIP and EDC) and flexor superficialis (FDS) muscles. The absence of a decelerating burst in the extensors and the relative silence of the flexor digitorum profundus (FPD) is sensitive to moment arm parameters. Future work will establish whether EMG signals measured in human subjects [8] are compatible with this optimal control policy. Our advances enable the testing of these hypotheses in future work because it demonstrates a first example of the scalability and applicability of the iLQG framework to complex nonlinear neuromuscular systems.

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